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## An Analytical Method for Optimal Capacitors Placement from the Inversed Reduced Jacobian Matrix

Ardiaty Arief<sup>a,\*</sup>, Antamil<sup>b</sup>, Muhammad Bachtiar Nappu<sup>a</sup>

<sup>a</sup> Department of Electrical Engineering, Universitas Hasanuddin, Makassar 90245, Indonesia

<sup>b</sup> Department of Electrical Engineering, Universitas Islam Makassar, Makassar 90245, Indonesia

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### Abstract

This paper presents a novel analytical methodology to determine location for optimal shunt capacitor placement in power systems. The new method modifies modal analysis technique and develops new formulation to compute the Reactive Contribution Index (RCI) of each load buses based on the inversed reduced Jacobian matrix. The objective of this study is to achieve the most stable condition as well as to minimize network losses. The proposed method is implemented at the modified IEEE 30-bus Reliability Test System (RTS). The simulation results show the proposed method can provide a solution to the ideal shunt capacitor placement to improve the system's voltage stability.

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*Keywords:* capacitor placement; inversed reduced Jacobian matrix; losses reduction; modal analysis; voltage stability.

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### 1. Introduction

Nowadays, modern power systems are severely stressed and working at the stability limit with smaller capacity and margin hence may cause congestion problems [1-5]. The progressive and uncontrollable drop in voltage as a result of increase in load demand, especially reactive loads or change in system operation conditions could eventually result in a wide spread voltage collapse [6-8]. Therefore protective step may be taken, such as load shedding [9-11].

To avoid this instability, there are several preventive step that can be done, which one of them is installation of shunt capacitor as reactive power compensation scheme. Shunt capacitor in power system provides reactive power compensation, reduces network losses, decreases energy losses, improves voltage profile, releases system capacity and recovers power factor [12-14]. Nevertheless, appropriate size and number of capacitors are required to maintain system stability that corresponds to the needs of the load in the system. In addition, the placement must be precise so that all critical areas can operate at a stable voltage range. Capacitor size must also be efficient to improve the voltage profile of the system to be at least above the threshold point of voltage stability.

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\* Corresponding author. Tel.: +62-812-41-693-305.

E-mail address: [ardiaty@eng.unhas.ac.id](mailto:ardiaty@eng.unhas.ac.id)

This study developed a new method that improve modal analysis [15] for the effective placement of shunt capacitors. In modal analysis method, the inversed reduced Jacobian matrix is computed that informs relation between change in voltage and change in reactive power injection at each buses. By utilizing the relationship of V and Q, this approach can identify weak points and areas that are vulnerable to system stability. The relationship between voltage (V) and reactive power (Q) can be used to obtain useful information of stability.

This study further formulates new Reactive Contribution Index (RCI) that is obtained from the inversed reduced Jacobian matrix. RCI gives information about contribution of a specific bus in improving voltage magnitude at critical buses. The bus with the biggest RCI has the most influence in improving the stability in the critical areas hence is chosen as the location of shunt capacitor placement. This method can be used as a solution for the installation of reactive power compensation. The advantage with this approach is fast, straight, accurate and do not involve composite computational processes.

**2. The proposed methodology**

To perform modal analysis, linearized steady-state power voltage equations is used as follow:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_{P\theta} & J_{PV} \\ J_{Q\theta} & J_{QV} \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix} \tag{1}$$

Voltage stability is strongly influenced by the active power and reactive power. Since this study focuses on the reactive power compensation, therefore the active power (P) is kept constant in order to see the relationship of voltage and reactive power stability. From elaboration of modal analysis, we obtain:

$$\Delta V = J_R^{-1} \Delta Q \tag{2}$$

Where,

$$J_R = [J_{QV} - J_{Q\theta} J_{P\theta}^{-1} J_{P\theta}]$$

$J_R^{-1}$  or the inversed reduced Jacobian matrix is a matrix that shows direct correlation between the bus voltage and reactive power injection. Assuming the active power is zero and the voltage angle is substituted, then obtained from the equation is much simpler and more focused.

From Eq. 2, it can be seen that the voltage change ( $\Delta V$ ) is obtained by multiplying the reactive power injection at specific bus with the inversed reduced Jacobian matrix ( $J_R^{-1}$ ). When the  $J_R^{-1}$  is further elaborated, then,

$$J_R^{-1} = \begin{bmatrix} \mathfrak{R}_{11} & \mathfrak{R}_{12} & \dots & \mathfrak{R}_{1n} \\ \mathfrak{R}_{21} & \mathfrak{R}_{22} & \dots & \mathfrak{R}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \mathfrak{R}_{m1} & \mathfrak{R}_{m2} & \dots & \mathfrak{R}_{mn} \end{bmatrix} \tag{3}$$

The RCI values at each load buses are computed by adding the elements of inversed reduced Jacobian matrix vertically, so that RCI at bus j becomes,

$$RCI_j = \sum_{i=1}^n \mathfrak{R}_{ij} \tag{4}$$

Where,

$\mathfrak{R}_{ij}$  is the  $i^{th}$  row and  $j^{th}$  column element of the inversed reduced Jacobian matrix.

RCI informs the contribution of each bus in improving voltage stability. Furthermore, as the aim of capacitor placement is to improve voltage stability, therefore the computation of RCI is focusing only on the effect to the unstable buses. The RCI is calculated at all buses in respect to the unstable buses. So that, to ensure all the buses voltage are stable then this method is done with few iterations by adding reactive power compensation until all the voltage are at the stable value. Once the bus voltage is becomes stable, it is excluded from the next process of RCI calculation.

**3. Test results and analysis**

To conduct a comprehensive test of the above methods, IEEE 30-bus Reliability Test System (RTS) is used. This test system is modified in such a way to obtain an initial state that allows it to perform testing of the proposed method.

Fig. 1 shows the system voltage profile on the initial conditions. The total active power losses are 23.383 MW, while the reactive power losses are 51.261 MVar.

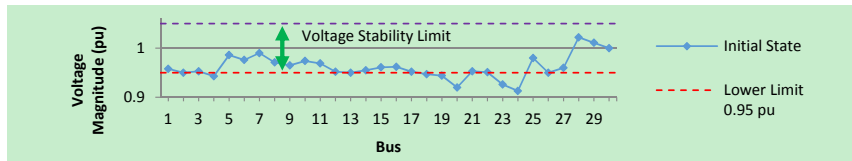


Fig. 1. Voltage magnitude at the initial state

As can be seen in Fig. 1, there are six critical load buses whose voltages are below 0.95 pu voltage stability limit, that are bus 4, 18, 19, 20, 23 and 24. To determine the right location for capacitor placement for covering the shortage of reactive power, then the RCI is calculated at all buses to observe their impacts to these critical buses. Therefore with 6 unstable buses, RCI at each load buses is calculated in respect of the critical buses, hence,

$$RCI_j = \mathfrak{R}_{4,j} + \mathfrak{R}_{18,j} + \mathfrak{R}_{19,j} + \mathfrak{R}_{20,j} + \mathfrak{R}_{23,j} + \mathfrak{R}_{24,j}$$

At the first iteration, bus 24 has the highest RCI, 1.9542. Hence bus 24 become the best location for capacitor placement for the first iteration.

After the first iteration of power flow, with an injection of 5 MVar, voltage magnitude at buses around bus 24 improve. The voltages at buses close to the bus 24 such as bus 21 and bus 23 straight increases significantly. However, the voltage at buses 24, 18, 19 and 20 are still below the stability limit 0.95 pu, then reactive compensation injection is still need to be added and the RCI is calculated again to find the next location for reactive compensation placement. In the second iteration, RCI at bus 20 is the highest. With another addition of reactive compensation at bus 20 of 5 MVar, the system is not stable yet, hence another process of calculating RCI is done again at the third iteration and found that bus 4 has the highest RCI. With 5 MVar capacitor at bus 4, the system has recovered its stability, hence the process is stopped. Fig. 2 informs the computation of RCI at all buses.

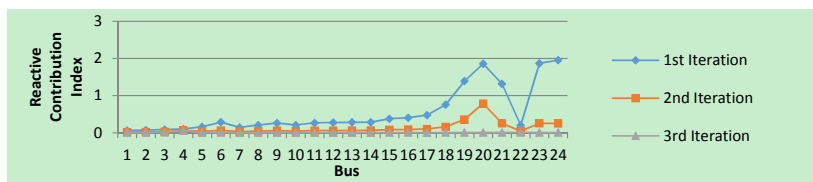


Fig. 2. Reactive Contribution Index (RCI) computation iteration process

Fig. 3 shows the voltage profile improvement at each iteration after capacitor placement at the nominated buses. Total capacitors required is 15 MVar with the assumption that the minimum voltage is achieved on each bus that is injected is 0.95 pu. After the installation of the 3 node above, then the voltage profile of the system has recovered to its stable condition. Buses that were previously under voltage stability limit is now above 0.95 pu.

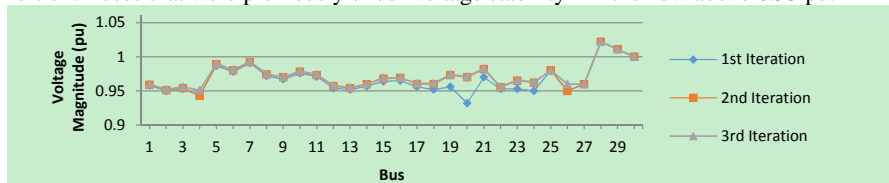


Fig. 3. Voltage profile improvement at each iteration

#### 4. Conclusions

A new method of capacitor placement by utilizing a direct connection between V & Q in the inversed reduced Jacobian matrix is proposed. A new Reactive Contribution Index (RCI) is formulated which is computed from the elements of the inversed reduced Jacobian matrix. The RCI informs how big the contribution of a specific bus in improving voltage magnitude at critical buses. The bigger the RCI of a bus, the bigger the influence of that bus in improving the voltage of the critical buses. The proposed method is tested on the IEEE 30-bus reliability test system. Based on the proposed method, the total reactive power compensator for the system 15 MVar.

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